STUDYING ONLINE PARTICIPATION AND ITS IMPACT ON THE DEVELOPMENTOF MATHEMATICAL CONTENT KNOWLEDGE FOR TEACHING

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We report on our efforts to support teachers' development of mathematical knowledge for teaching through online professional development. In particular, we report on our investigation into the relationship between online interaction and teachers' development of mathematics content for teaching. Through the integration of content analysis and social network analysis, we identify underlying relationships between aspects of online interaction and teacher learning. Results indicate that while interaction, broadly speaking, was not correlated with teacher learning, particular combinations of content and the centrality of an individual in the interaction were. Implications of these significant correlations for mathematics teacher education are discussed.

For more then a decade, the importance of teachers' content knowledge has been a major focus in the literature (Ball, 1993; Ma, 1999; Shulman, 1986) and increasing teachers' mathematical knowledge continues to be a major focus in both education research and policy (Greenberg & Walsh, 2008; National Mathematics Advisory Panel, 2008). Despite these calls, a great number of elementary teachers continue to be underprepared and uncomfortable with the mathematics content they are expected to teach (Greenberg & Walsh, 2008). At the secondary level, studies have shown that students who have been successful in high school and university mathematics classes – even those with grades of A in calculus – often have weak or underdeveloped understandings of the concept of function, a core concept in the K-12 mathematics curriculum (Carlson, Jacobs, Coe, Larsen, & Hsu, 2002; Monk, 1992; Thompson, 1994). With regards to the mathematics preparation of school teachers, Cuoco (2001) notes the solutions "will not come from rearranging the topics in a syllabus or by adding more topics to an already bloated undergraduate curriculum. Making lists of topics that teachers should know … won't do it either" (p. 170). He argues that teachers need experiences *doing* mathematics and to develop a "taste" and passion for it. Bass (2005) notes that regardless of their level mathematical preparation, evidence suggests that that teachers lack *Mathematical Knowledge for Teaching* (MKT): "the mathematical knowledge, skills, habits of mind, and sensibilities that are entailed by the actual work of teaching" (p 429).

Throughout the past two years, we have engaged in an extended research project focusing on supporting teachers' mathematical content knowledge and mathematical knowledge for teaching through online professional development. The overarching goal has been to support teachers' mathematical development through authentic engagement in collaborative mathematical problem solving and connecting that engagement with the teaching of school mathematics. In this paper, we report on the results of a subset of this project that sought to support teachers' developing a coherent view of the school algebra curriculum and mathematics content knowledge for teaching school algebra. The primary research question we explore is *What is the relationship between online participation in teacher development activities and teachers' development of MKT?*

Theoretical Background

The majority of online instruction emphasizes the importance of interaction and the affordances of the internet in supporting interaction at a distance, but pays little attention to the key features and functions of those interactions. We have found Lotman's (1988) characterizations of text as *univocal* or *dialogic* particularly useful in understanding the function of the individual posts that make up an online interaction and the interaction as a whole. Univocal discourse functions "to convey meaning adequately" (Lotman, 1988, p. 34). In contrast, dialogic discourse can be characterized as a "thinking device." Lotman describes dialogic function as generating new meaning: "In this respect a text ceases to be a passive link in conveying some constant information between input (sender) and output (receiver). … [I]n its second function a text is not a passive receptacle, or bearer of some content placed in it from without, but a generator" (Lotman, 1988, pp. 36-37). This dualism has guided both the design of our teacher development activities and our analysis of the online interactions.

In our work with teachers, we seek to provide environments that support the dialogic function of text, where teachers view and use each others posts as "thinking devices." We, then, attempt to understand these interactions in terms of their potential and success for catalyzing the generation of new mathematical knowledge and knowledge for teaching. This perspective is consistent with a variety of research in the learning sciences that emphasizes the importance of discourse and interaction in learning (Mercer, 2000; Paloff & Pratt, 1999; Ravenscroft, 2001; Shale & Garrison, 1990; Su, Bonk, Magjuka, Liu, & Lee, 2005). Previous results documenting the relationship between interaction and learning include that learning is promoted through dialogue (Cunningham, 1992) and that learning occurs online through social activity in communities (Haythornthwaite, 2002; Rovai, 2002).

Through a design research methodology, we have developed and explicated a model for Online Asynchronous Collaboration (OAC) in mathematics teacher education, which has at its core cycles of individual, small group, and whole class interaction (see Silverman & Clay, 2010 for a detailed discussion of OAC). OAC begins with participants drafting a solution, initial approach, or questions on a set of purposefully selected, open-ended mathematics tasks in a private online workspace. In these initial posts, we do not expect that each teacher will be able to complete each of these activities, but we do expect each participant to attempt the assigned task and either pose a solution method and solution or ask relevant questions or wonderings that reflect their current state of thinking about the task. After the individual work phase, each participant's initial postings are made public for small-group and dyadic interactions where participants add comments, respond to classmates' questions, or ask for clarifications. Finally, the instructors orchestrate a whole-class discussion that lies at the confluence of the participants' collective sense making and the sessions' instructional objectives.

We have had success documenting participants' mathematical development (Clay & Silverman, 2008; Silverman & Clay, 2009) and the role of the teacher in supporting that development (Clay & Silverman, 2009). Despite these successes, we have struggled to identify correlations between the online teacher development activity and teacher development. In this paper, we focus our attention explicitly on this relationship.

Setting and Participants

In this article, we focus on an online graduate class in mathematics education that focused specifically on proportional and algebraic reasoning. The course was designed to support teachers as they deepen and extend their mathematical understandings and develop schemas within which a variety of mathematical ideas are conceptually connected (Silverman & Clay,

2009). First, we began by exploring mathematical ideas that the teachers teach regularly and ostensibly know well. We then sought to problematize the teachers' current mathematical understandings through posing "changes" to the mathematical idea (for example the exclusion, replacement or addition of properties, generalization, application to a new setting, etc.). These new conceptualizations of school math concepts were then used to support instructional conversations that bridged teachers' developing understandings and the school mathematics curriculum and focused on how various mathematical ideas fit together.

The wholly online class took place in the winter term of 2008 at an urban research university in the Northeastern United States. The class consisted of 17 (13 women, 4 men) who were geographically distributed throughout the United States. The 10-week course was hosted in the Blackboard Learning Management System. Within Blackboard, participants accessed course materials (readings and activities) from their homes or workplaces and participated in both threaded discussion boards and blogs. Each week, participants were presented with prompts to consider when creating their posts, but were consistently reminded that the prompts were meant as guides and not as a list of questions that must be answered. For each discussion board, participants were required to post a minimum of one initial post by Tuesday evening and two replies by Saturday evening.

Data Sources and Analytical Procedures

The two primary sources of data for this study include online interaction data, taken from the participants' interaction in the online discussion forums, and teacher learning data. Teacher learning was measured using instruments developed and validated by the Learning Mathematics for Teaching Project (http://sitemaker.umich.edu/lmt) for measuring teachers *Mathematical Knowledge for Teaching.* For this paper, the two versions of the LMT instrument for patterns, functions, and algebra were administered and are used as a validated measure of participant learning of the relevant content. Teachers' gain scores were converted to Item Response Theory (IRT) scaled scores using conversion tables provided by the developers.

Coding and Content Analysis

While raw interaction data (who talked to whom) is relatively easy to generate and can provide us with useful information about the interactions between teachers, this information does not provide us with any information about the content of and function of those interactions. In essence, it neglects the basic question posed by Schlager, Farooq, Fusco, Schank & Dwyer (2009): "What constitutes a meaningful relation or tie between individuals?" (p. 87). In order to better understand this question, we employed a variant of content analysis to compare, contrast, and categorize the posts. Coding of interaction data began with Lowes, Lin & Lang's (2007)'s coding scheme for coding interactivity in an online class, which was adapted to meet the needs of this project through a recursive process. Ultimately, the following codes became stable for our data set:

Cheerleading/Affirming (C/A)

- Offering praise and encouragement
- Focus on ideas not related to mathematics or teaching (ie. use of learning management system, course logistics, personal issues, etc.)
- Affirmation of work without any evidence of reflection

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Students, Teaching, and Schools (STS)

- New information about students and teaching from teachers' own classroom or classroom experience

Direct Instruction (DI)

- Providing direct instruction to classmates; telling how to do or solve a problem

Doing Mathematics (DM)

- Making mathematical activity (thinking, representing, conjecturing, etc.) public

Questioning/ Challenging (Q/C)

- Raise questions that extend previous post, or express disagreement with it

All posts were coded on each of the two dimensions by two researchers using the above scheme. In order to ensure the reliability of the coding process, the researchers coded the first 30 posts together and came to agreement on the codes, definitions, and examples. The researchers then coded the entire data set independently and approximately 77% of the items were coded similarly by both researchers. Each item that was coded differently was discussed and deliberated until agreement was reached on how to code the item in question.

Social Network Analysis

Social network analysis (SNA), a mathematical approach for analyzing interactions and the structure of social network and the strength of the ties between actors in the network (Wasserman & Faust, 1994), was used to quantify the interactions being studied. One primary use of social network analysis is to identify important actors in the interaction and we use these measures of importance to further parse the online interaction data and to quantify teachers' role in the online interaction. Within social network analysis, there are two primary ways of quantifying the "importance" of actors in an interaction: centrality and prestige. An actor's centrality, or the number of posts generated by the actor is a measure of his or her influence on others within the social environment. In particular, it is the actor's potential influence on the interaction. When one has a high centrality, he or she is in direct contact with many others: "this actor should then be recognized by others as a major channel of relational information … a crucial cog in the network, occupying a central location" (Wasserman & Faust, 1994, p. 179). An actor's degree prestige, or the number of posts received by the actor (the in-degree), is a measure of how he or she is potentially influenced by others in the social environment. In terms of online learning, an actor's prestige can be interpreted as the amount to which colleagues (other actors) seek out and support an individual (Russo & Koesten, 2005).

Analysis of Relationship Between Online Participation and Increases in Mathematical Knowledge for Teaching

In order to quantify participants' participation, each participant's centrality and prestige was calculated (both their overall centrality and prestige as well as centrality and prestige broken down by each of the codes discussed above). Finally, Pearson's correlation was used to identify statistically significant linear relationships between participants' centrality and prestige and gains in mathematical knowledge for teaching, as measured by the LMT instruments.

Results

Analysis of the relationship between individual participation and LMT gain scores was conducted on the twelve independent variables shown below in Table 1.

Table 1: Twelve Independent Variables

	Centrality	**Prestige**
Overall	[1]	[2]
Cheerleading/Affirming	[3]	[4]
Students, Teaching and Schools	[5]	[6]
Direct Instruction	[7]	[8]
Doing Mathematics	[9]*	[10]
Questioning/Challenging	[11]*	[12]

An analysis of the linear relationship between each of the independent variables for participants' content prestige and centrality and participant learning was conducted using Pearson's correlation coefficient. This analysis indicated the following positive relationships (indicated by * in Table 1 above):

- A positive relationship between participants' Doing Mathematics (DM) centrality and their LMT gain score, $r(15) = 0.624$, $p < 0.05$.
- A positive relationship between participants' Questioning and Challenging (Q/C) centrality and their LMT gain score, $r(15) = 0.631$, $p < 0.05$.

Conclusion and Discussion

In this study, we bring into question the common belief and research results that emphasize the importance of interaction in online learning (Paloff & Pratt, 1999; Shale & Garrison, 1990; Su et al., 2005). In particular, we present evidence for the claim that it is not *simply* interaction that is important for learning in an online context. We extend Schlager, Farooq, Fusco, Schank & Dwyer's (2009) question of "What constitutes a meaningful relation or tie between individuals?" (p. 87) in two key directions: the content of the contribution and the individuals' patterns of activity. Results indicate that while overall levels of activity in online interactions were not correlated with gains in mathematical knowledge for teaching, the centrality of individuals Doing Mathematics and Questioning/Challenging were. Put another way, teachers with high activity (ie. teachers who generated a large number of posts) coded as Doing Mathematics and Questioning/Challenging tended to have higher LMT gain scores.

We recognize that this result might seem counterintuitive: one of the benefits of social networks (like online classes – or schools, for that matter) is that individuals can learn and grow through individual interactions with group members. For example, research has shown that *who* a teacher "gets help from" and not *the amount* of help a teacher gets was related to changes in teaching practices (Penuel & Riel, 2007). In other words, it seems that the posts directed towards an individual should be an important predictor of teacher learning. This was not the case in this study: participants' learning was correlated with the centrality of an individual (the number of posts an individual directed towards others), provided the posts involved Doing Mathematics or Questioning/Challenging.

We reconcile this apparent contradiction by focusing on the nature of MKT. We have argued previously that the development of MKT starts with coherent mathematical understandings and involves personal reconstruction of one's mathematical understandings to include both (1) the variety of ways that individuals may understand the idea and (2) how particular ways of understanding can empower individuals to learn other, related mathematical ideas (Silverman & Thompson, 2008). We also argued that this personal reconstruction is necessarily the result of a

reflecting abstraction (Steffe, 1991) and not easily developed through explanation or demonstration: "the transition requires a building up of the understanding through students' activity and reflection and usually comes about over multiple experiences" (Simon, 2006, p. 362). Framed this way, it appears to make sense that our participants' learning was not correlated with explanations and demonstrations directed towards the participant (others' posts directed to the participant). Doing Mathematics and Questioning/Challenging centrality can be interpreted as a metric for the one's level of personal engagement and reflection in interactions designed to support the development of MKT. We speculate that participants with a high Cheerleading or Direct Instruction are experiencing the same discussion board quite differently than those with high Doing Mathematics and Questioning/Challenging centrality: rather than "telling" they are "doing" and "wondering" and placing themselves in a state of cognitive disequilibrium, which has the possibility of stimulating equilibration (learning).

Implications for Mathematics Teacher Education

The primary implication for mathematics teacher education is that, while teachers need to be challenged and encouraged to engage with their colleagues as part of their professional development activities, efforts need to be made to increase the amount of Doing Mathematics and Questioning/Challenging that they engage in. One obvious method, which we have implemented with little success, is being more specific about the posting requirements (for example, requiring at least one Doing Mathematics or Questioning/ Challenging post). We believe that there are two reasons for our lack of significant results from more specific requirements: our past participants (1) want to talk about what they believe their current needs to be and doing or learning math is often not a current perceived need and (2) often believe that direct instruction *is* the essence of doing mathematics. With these two factors in mind, it is clear that the challenge is not one of imposing requirements but rather one of a cultural shift: teachers need to come to see the value of MKT for their teaching and that the more teachers engage in *doing* mathematics, the more effective they will be at supporting their students as they do mathematics. Thus, it seems that one way to increase the effectiveness of online implementations designed to foster MKT would be focused teacher development activities designed to support this cultural shift. Examples of such activities include the Math Forum's Online Mentoring Project (Shumar, 2006) and dialogue games (Ravenscroft, 2007).

Future Work

We fully recognize that all of the emphasis cannot be placed on increasing participants' Doing Mathematics and Questioning and Challenging centrality – these posts need not and do not happen in a vacuum. A great many Doing Mathematics posts were in response to others' posts and it is necessary to have something (a post) to question or challenge. Further, we acknowledge that other types of posts (cheerleading, for example) may be useful in supporting generative discourse. We have begun to explore sequential analysis (Bakeman & Gottman, 1997; England, 1985; Jeong, 2003) to study group interactions. Sequential analysis involves using probability to study relationships between individual posts and predict the characteristics of interactions that support individual contributions with particular characteristics (for example, "Doing Mathematics" posts) and the conditions under which those posts are more likely. In addition to increasing the effectiveness of our online teacher development, we believe that this research can help us bridge our work and the current emphasis on online communities in teacher professional development.

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